

5-5

Direct Variation

Lesson Preview

What You'll Learn

OBJECTIVE 1 To write an equation of a direct variation

OBJECTIVE 2 To use ratios and proportions with direct variations

... And Why

To write a direct variation relating to weather, as in Example 3

Check Skills You'll Need

(For help, go to Lessons 2-6 and 4-1.)

Solve each equation for the given variable.

1. $nq = m; q$

2. $d = rt; r$

3. $ax + by = 0; y$

Solve each proportion.

4. $\frac{5}{8} = \frac{x}{12}$

5. $\frac{4}{9} = \frac{n}{45}$

6. $\frac{25}{15} = \frac{y}{3}$

7. $\frac{7}{n} = \frac{35}{50}$

8. $\frac{8}{a} = \frac{20}{36}$

9. $\frac{14}{18} = \frac{63}{n}$

New Vocabulary • direct variation • constant of variation

OBJECTIVE



Interactive lesson includes instant self-check, tutorials, and activities.

1 Writing the Equation of a Direct Variation

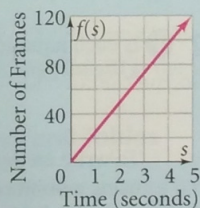
Investigation: Direct Variation

As you watch a movie, 24 individual pictures, or frames, flash on the screen each second. Here are three ways you can model the relationship between the number of frames $f(s)$ and the number of seconds s .

Table

s number of seconds	$f(s)$ number of frames
1	24
2	48
3	72
4	96
5	120

Graph



Function Rule

$$f(s) = 24s$$

- As the number of seconds doubles, what happens to the number of frames?
- Find the ratio $\frac{\text{number of frames}}{\text{number of seconds}}$ for each pair of data in the table.
- For every increase of 1 second on the horizontal axis of the graph, what is the increase on the vertical axis?
- What do you notice about your answers to Questions 2 and 3 and the coefficient of s in the function rule?
- What number of frames corresponds to $s = 0$?
 - What is the ordered pair on the graph for the seconds and number of frames when $s = 0$?

When a film is projected the number of frames doubles as the number of seconds doubles. The number of frames is proportional to the number of seconds; that is, the number of frames varies directly with the number of seconds.

Key Concepts

Definition Direct Variation

A function in the form $y = kx$, where $k \neq 0$, is a **direct variation**. The **constant of variation** k is the coefficient of x . The variables y and x are said to vary directly with each other.

Reading Math

Constant means *remaining the same*. Constant of variation means changing at the *same rate*.

For $y = kx$, y is a function of x . If $x = 0$, then $y = 0$, so the graph of a direct variation is a line that passes through $(0, 0)$. To tell whether an equation represents a direct variation, solve for y . If the equation can be written in the form $y = kx$, it represents a direct variation.

1 EXAMPLE Is an Equation a Direct Variation?

Is each equation a direct variation? If it is, find the constant of variation.

a. $5x + 2y = 0$

$$2y = -5x \quad \text{Subtract } 5x \text{ from each side.}$$

$$y = -\frac{5}{2}x \quad \text{Divide each side by 2.}$$

The equation has the form $y = kx$, so the equation is a direct variation.

The constant of variation is $-\frac{5}{2}$.

b. $5x + 2y = 9$

$$2y = 9 - 5x \quad \text{Subtract } 5x \text{ from each side.}$$

$$y = \frac{9}{2} - \frac{5}{2}x \quad \text{Divide each side by 2.}$$

The equation cannot be written in the form $y = kx$. It is not a direct variation.

Check Understanding

1 Is each equation a direct variation? If it is, find the constant of variation.

a. $7y = 2x$

b. $3y + 4x = 8$

c. $y - 7.5x = 0$

To write an equation for a direct variation, you first find the constant of variation k using a point other than the origin that lies on the graph of the equation. Then use the value of k to write an equation.

2 EXAMPLE Writing an Equation Given a Point

Write an equation of the direct variation that includes the point $(4, -3)$.

$$y = kx \quad \text{Start with the function form of a direct variation.}$$

$$-3 = k(4) \quad \text{Substitute 4 for } x \text{ and } -3 \text{ for } y.$$

$$-\frac{3}{4} = k \quad \text{Divide each side by 4 to solve for } k.$$

$$y = -\frac{3}{4}x \quad \text{Write an equation. Substitute } -\frac{3}{4} \text{ for } k \text{ in } y = kx.$$

An equation of the direct variation is $y = -\frac{3}{4}x$.

Check Understanding

2 Write an equation of the direct variation that includes the point $(-3, -6)$.



Real-World Connection

The total energy released by a single flash of lightning could power an ordinary light bulb for a few months.

Check Understanding

OBJECTIVE

2 Ratios, Prop



Real-World Connection

The total energy released by a single flash of lightning could power an ordinary light bulb for a few months.

You can use a direct variation to describe a real-world situation in which the dependent variable varies directly with the independent variable.

3 EXAMPLE Real-World Problem Solving

Weather Your distance from lightning varies directly with the time it takes you to hear thunder. If you hear thunder 10 seconds after you see lightning, you are about 2 miles from the lightning. Write an equation for the relationship between time and distance.

Relate The distance varies directly with the time. When $x = 10$, $y = 2$.

Define Let $x =$ the number of seconds between your seeing lightning and your hearing thunder.

Let $y =$ your distance in miles from the lightning.

Write $y = kx$ Use the general form of a direct variation.

$2 = k(10)$ Substitute 10 for x and 2 for y .

$\frac{1}{5} = k$ Divide each side by 10 to solve for k .

$y = \frac{1}{5}x$ Write an equation. Substitute $\frac{1}{5}$ for k in $y = kx$.

The equation $y = \frac{1}{5}x$ relates the time x in seconds it takes you to hear the thunder to the distance y in miles you are from the lightning.

Check Understanding

3 A recipe for a dozen corn muffins calls for 1 cup of flour. The number of muffins varies directly with the amount of flour you use. Write a direct variation for the relationship between the number of cups of flour and the number of muffins.

OBJECTIVE

2 Ratios, Proportions, and Direct Variations

You can rewrite a direct variation $y = kx$ as $\frac{y}{x} = k$. When two sets of data vary directly, the ratio $\frac{y}{x}$ is the constant of variation. It is the same for each data pair.

4 EXAMPLE Direct Variations and Tables

For each table, use the ratio $\frac{y}{x}$ to tell whether y varies directly with x . If it does, write an equation for the direct variation.

a.

x	y	$\frac{y}{x}$
-3	2.25	$\frac{2.25}{-3} = -0.75$
1	-0.75	$\frac{-0.75}{1} = -0.75$
4	-3	$\frac{-3}{4} = -0.75$
6	-4.5	$\frac{-4.5}{6} = -0.75$

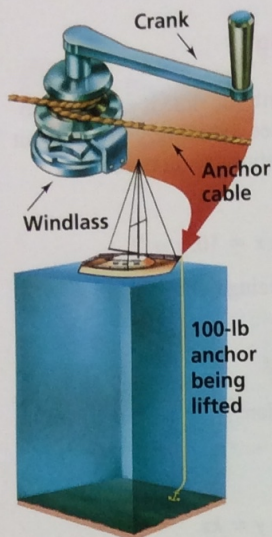
Yes, the constant of variation is -0.75 . The equation is $y = -0.75x$.

b.

x	y	$\frac{y}{x}$
2	-1	$\frac{-1}{2} = -0.5$
4	1	$\frac{1}{4} = 0.25$
6	3	$\frac{3}{6} = 0.5$
9	4.5	$\frac{4.5}{9} = 0.5$

No, the ratio $\frac{y}{x}$ is not the same for all pairs of data.

Check Understanding



A windlass is a winch turned by a crank. It is used in a water well and to raise an anchor on a boat.

4 For the data in each table, tell whether y varies directly with x . If it does, write an equation for the direct variation.

a.

x	y
-2	3.2
1	2.4
4	1.6

b.

x	y
4	6
8	12
10	15

In a direct variation, the ratio $\frac{y}{x}$ is the same for all pairs of data where $x \neq 0$. So the proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ is true for the ordered pairs (x_1, y_1) and (x_2, y_2) , where neither x_1 nor x_2 are zero.

5 EXAMPLE

Real-World Problem Solving

Physics The force you must apply to lift an object varies directly with the object's weight. You would need to apply 0.625 lb of force to a windlass to lift a 28-lb weight. How much force would you need to lift 100 lb?

Relate A force of 0.625 lb lifts 28 lb. What force lifts 100 lb?

Define Let n = the force you need to lift 100 lb.

Write $\frac{\text{force}_1}{\text{weight}_1} = \frac{\text{force}_2}{\text{weight}_2}$ Use a proportion.

$$\frac{0.625}{28} = \frac{n}{100} \quad \text{Substitute 0.625 for force}_1, 28 \text{ for weight}_1, \text{ and } 100 \text{ for weight}_2.$$

$$0.625(100) = 28n \quad \text{Use cross products.}$$

$$n \approx 2.2 \quad \text{Solve for } n.$$

• You need about 2.2 lb of force to lift 100 lb.

Check Understanding

5 **Physics** Suppose a second windlass requires 0.5 lb of force to lift an object that weighs 32 lb. How much force would you need to lift 160 lb?

EXERCISES

Practice and Problem Solving

For more practice, see *Extra Practice*.

A Practice by Example

Example 1
(page 262)

Is each equation a direct variation? If it is, find the constant of variation.

1. $2y = 5x + 1$

2. $8x + 9y = 10$

3. $-12x = 6y$

4. $y + 8 = -x$

5. $5x - 6y = 0$

6. $-4 + 7x + 4 = 3y$

7. $-x = 10y$

8. $0.7x - 1.4y = 0$

9. $\frac{1}{2}x + \frac{1}{3}y = 0$

Example 2
(page 262)

Write an equation of the direct variation that includes the given point.

10. (1, 5)

11. (5, 1)

12. (-8, 10)

13. (-5, -9)

14. (-2, 3)

15. (-6, 1)

16. (3, -4)

17. (6, -8)

18. (-6, 8)

19. (-5, -10)

20. (12, -8)

21. (35, 7)



Real-World You must be at least old and weigh at least pounds to give blood.

Example 3
(page 263)

Define the variables. Then write a direct variation to model each relationship.

22. **Geometry** The perimeter of a regular octagon varies directly with the length of one side of the octagon.
23. **Earnings** When you have a job that pays an hourly wage, the amount you earn varies directly with the number of hours you work. Suppose you earn \$7.10/hour working at the library.

Example 4
(page 263)

For the data in each table, tell whether y varies directly with x . If it does, write an equation for the direct variation.

24.

x	y
3	5.4
7	12.6
12	21.6

25.

x	y
-2	1
3	6
8	11

26.

x	y
-6	9
1	-1.5
8	-12

Example 5
(page 264)

27. **Physics** The maximum weight you can lift with a lever varies directly with the amount of force you apply. Suppose you can lift a 50-lb weight by applying 20 lb of force to a certain lever.
- What is the ratio of weight to force for the lever?
 - Write a proportion and find the force you need to lift a friend weighing 130 lb.

28. **Bicycling** A bicyclist traveled at a constant speed during a timed practice period. Write a proportion to find the distance the cyclist traveled in 30 min.

A Bicyclist's Practices

Elapsed Time	Distance
10 min	3 mi
25 min	7.5 mi

B Apply Your Skills

Write an equation of the direct variation that includes the given point.

29. $(3, \frac{1}{2})$ 30. $(\frac{1}{4}, -5)$ 31. $(-\frac{5}{6}, \frac{6}{5})$ 32. $(1.2, 7.2)$
 33. $(0.5, 4.5)$ 34. $(-2, \frac{1}{16})$ 35. $(5.2, -1.5)$ 36. $(-\frac{8}{3}, -\frac{9}{8})$

37. **a. Writing** How can you tell whether two sets of data vary directly?
b. How can you tell if a line is the graph of a direct variation?

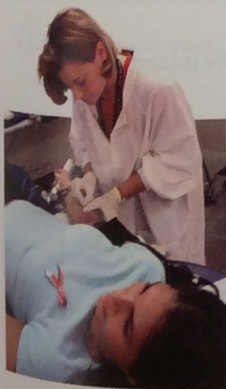
Critical Thinking Is each statement true or false? Explain.

38. The graph of a direct variation may pass through $(-2, 4)$.
 39. The graph of a direct variation may pass through $(0, 3)$.
 40. If you triple an x -value of a direct variation, the y -value also triples.

Graph the direct variation that includes the given point.
 Write an equation of the line.

41. $(2, 5)$ 42. $(-2, 5)$ 43. $(2, -5)$ 44. $(-2, -5)$

45. **Biology** The amount of blood in a person's body varies directly with body weight. A person who weighs 160 lb has about 5 qt of blood.
- Find the constant of variation.
 - Write an equation relating quarts of blood to weight.
 - Open-Ended** Estimate the number of quarts of blood in your body.



Real-World Connection

You must be at least 17 years old and weigh at least 110 pounds to give blood.

46. **Electricity** Ohm's Law $V = I \times R$ relates the voltage, current, and resistance of a circuit. V is the voltage measured in volts. I is the current measured in amperes. R is the resistance measured in ohms.
- Find the voltage of a circuit that has a current of 24 amperes and resistance of 2 ohms.
 - Find the resistance of a circuit that has a current of 24 amperes and a voltage of 18 volts.

C Challenge

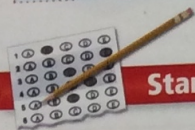
The ordered pairs in each exercise are for the same direct variation. Find each missing value.

47. (3, 4) and (9, y) 48. (-1, 2) and (4, y) 49. (-5, 3) and (x , -4.8)
50. (1, y) and ($\frac{3}{2}$, -9) 51. (2, 5) and (x , 12.5) 52. (-2, 5) and (x , -5)

Problem Solving Hint

For Exercise 53, start with the relationship of miles and gallons: $\frac{m}{g} = 24$.

53. **Gas Mileage** A car gets 24 miles per gallon. The number of gallons g of gas used varies directly with the number of miles m traveled.
- Suppose the price of gas is \$1.83 per gallon. Write a function relating the cost c for g gallons of gas. Is this a direct variation?
 - Write a direct variation relating the cost of gas to the miles traveled.



Standardized Test Prep

Multiple Choice

54. Which equation is a direct variation?
 A. $y = -0.7x$ B. $y = \frac{21}{x}$ C. $y - x = 4$ D. $y = 3x + 2$
55. A direct variation includes the point (-8, 2). Which is an equation of the direct variation?
 F. $-8y = x + 2$ G. $2y = -8x$ H. $y = \frac{x}{-4}$ I. $y = -4x$

Quantitative Comparison

Compare the boxed quantity in Column A with the boxed quantity in Column B. Choose the best answer.

- The quantity in Column A is greater.
- The quantity in Column B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

Use this statement for Exercises 56–58.

A direct variation includes the point (5, -4).

	Column A	Column B
56.	constant of variation of the equation	y -value, when $x = 3$
57.	y -value, when $x = 5$	y -value, when $x = -5$
58.	y -value, when $x = 0$	0

59. Write an equation of the direct variation that includes the point (-1, -4). Show your work.



Take It to the NET

Online lesson quiz at www.PHSchool.com
 Web Code: aea-0505

Short Response

Checkpoint

TEXT Instant se quiz onli on CD-RC

Mixed Review

Lesson 5-4

Write a function rule for each table.

60.

Number of People	Total Bill
1	\$3.00
2	\$6.00
3	\$9.00
4	\$12.00

61.

Amount Earned	Amount Spent
\$15	\$5
\$30	\$10
\$45	\$15
\$60	\$20

62.

Number of Days	Supplies Remaining
0	12 lb
2	10 lb
4	8 lb
6	6 lb

63.

Weight on Earth (lb)	Weight on Moon (lb)
96	16
123	20.5
144	24
171	28.5

Lessons 3-2, 3-3

Solve each inequality.

64. $r + 6 > -12$

65. $5 + c \leq 3.2$

66. $7m < -21$

67. $a - 4.5 \geq 12.1$

68. $\frac{n}{4} < -20$

69. $3r \geq 9.12$

70. $\frac{y}{-5} \leq \frac{1}{2}$

71. $b + 4\frac{2}{3} > 5\frac{1}{6}$

Lesson 1-6

72. **Shipping** For the ships that pass through the Panama Canal, the average toll is \$45,000 per ship. The canal authority earned about \$700 million in the year 2000. About how many ships passed through the canal that year? Round to the nearest hundred.

Checkpoint Quiz 2

Lessons 5-3 through 5-5

TEXT Instant self-check quiz online and on CD-ROM

Model each rule with a table of values and a graph. If the rule describes a direct variation, state the constant of variation.

1. $y = 4x + 1$

2. $y = \frac{1}{2}x$

3. $f(x) = -3x$

4. $y = -3x + 2$

Write a function rule for each situation.

5. the total cost $t(p)$ of p pounds of potatoes at \$.79 per pound

6. the total distance $d(n)$ traveled in n hours at a constant speed of 60 mi/h

Write an equation for the direct variation that includes the given point.

7. $(7, -2)$

8. $(-3, -6)$

9. $(-4, -5)$

10. **a. Bicycling** The distance a wheel moves forward varies directly with the number of rotations. Suppose the distance d the wheel moves is 56 ft when the number of rotations n is 8. Find the constant of variation and write a direct variation equation to model this situation.
b. Use the direct variation you wrote for part (a) to find the distance the wheel moves in 20 rotations.