Name: Key Class: _____

M8-U5: Notes & HW #7 – Practice Solving Systems

Date: _____

Example A:

A fashion designer makes and sells hats. The material for each hat costs $5.50. The hats sell for $12.50 each. The designer spends $1400 on advertising. How many hats must the designer sell to break even?

let: h = # of hats = 200
C = cost = $2500

$$\begin{cases} C = 5.50h + 1400 \\ C = 12.50h \end{cases}$$

$C = 5.50(200) + 1400 = \$2500$

$C = 12.50(200) = \$2500$ ✓

$12.50h = 5.50h + 1400$
$-5.50h \quad -5.50h$

$\frac{7h}{7} = \frac{1400}{7}$

$h = 200$

Example B:

A plane takes about 6 hours to fly you 2,400 miles from New York City to Seattle, Washington. At the same time, your friend flies from Seattle to New York City. His plane travels with the same average airspeed, but his flight takes 5 hours. Find the average airspeed of the planes. Find the average wind speed.

let: a = airspeed = 440mph
w = wind speed = 40mph

$d = rt$ or $r = \frac{d}{t}$

the wind helps, should take less time → $a + w = \frac{2400}{5}$

the wind is bad, takes longer → $a - w = \frac{2400}{6}$

$a + w = 480$
$a - w = 400$

$\frac{2a}{2} = \frac{880}{2}$

$a = 440$

$(440) + w = 480$
$-440 \quad -440$

$w = 40$

Solve the following systems algebraically, find the solution.

Tell whether the system has *no solution, one solution* or *infinitely many solutions.*

1. $\begin{cases} y = 3x - 10 \\ y = 2x - 5 \end{cases}$

$3x - 10 = 2x - 5$

$-2x \quad +10 \quad -2x \quad +10$

$x = 5$

$y = 3(5) - 10$

$= 15 - 10$

$y = 5$

$y = 2(5) - 5$

$= 10 - 5$

$y = 5$ ✓

$P(5,5)$ one solution

2. $\begin{cases} x = -2y + 1 \\ x = y - 5 \end{cases}$

$-2y + 1 = y - 5$

$+2y \quad +5 \quad +2y \quad +5$

$\frac{6}{3} = \frac{3y}{3}$

$2 = y$

$x = -2(2) + 1$

$= -4 + 1$

$x = -3$

$x = (2) - 5$

$x = -3$ ✓

$P(-3,2)$

3. $\begin{cases} 6x - 3y = 6 \\ y = 2x + 5 \end{cases}$

$6x - 3(2x + 5) = 6$

$-6x \quad -6x \quad -15 = 6$

$-15 \neq 6$

No solution

must be parallel.

4. $\begin{cases} y = 2x \\ 7x - y = 15 \end{cases}$

$7x - (2x) = 15$

$\frac{5x}{5} = \frac{15}{5}$

$x = 3$

$y = 2(3)$

$y = 6$

$P(3,6)$ one solution

CK

$7(3) - (6) \stackrel{?}{=} 15$

$21 - 6 \stackrel{?}{=} 15$

$15 = 15$ ✓

5. $\begin{cases} 4x+7y=3 \\ 4x+7y=5 \end{cases}$

based on inspection there is no solution

$4x+7y$ can't be equal to two different numbers & still have a solution.

6. $-\left(\begin{cases} x+3y=11 \\ 2x+3y=4 \end{cases}\right) \rightarrow \begin{array}{r} x+3y=11 \\ -2x-3y=-4 \end{array}$

$\frac{-x}{-1} = \frac{7}{-1}$

$\boxed{x=-7}$

$(-7)+3y=11$

$+7 \quad +7$

$\frac{3y}{3} = \frac{18}{3}$

$\boxed{y=6}$

$\boxed{P(-7,6) \text{ one solution}}$

Ck

$2(-7)+3(6) \stackrel{?}{=} 4$

$-14+18 \stackrel{?}{=} 4$

$4=4\checkmark$

7. $+\begin{cases} 6x-3y=3 \\ -6x+5y=3 \end{cases}$

$\frac{2y}{2} = \frac{6}{2}$

$\boxed{y=3}$

$6x-3(3)=3$

$6x-9=3$

$+9 \quad +9$

$\frac{6x}{6} = \frac{12}{6}$

$\boxed{x=2}$

$\boxed{P(2,3)}$

Ck

$-6(2)+5(3) \stackrel{?}{=} 3$

$-12+15 \stackrel{?}{=} 3$

$3=3\checkmark$

8. $8\left(7x-10y=-24\right)$

$-7\left(8x-7y=-23\right) \rightarrow$

$\begin{array}{r} 56x-80y=-192 \\ -56x+49y=161 \\ \hline -31y=-31 \end{array}$

$\frac{-31y}{-31} = \frac{-31}{-31}$

$\boxed{y=1}$

$7x-10(1)=-24$

$7x-10=-24$

$+10 \quad +10$

$\frac{7x}{7} = \frac{-14}{7}$

$\boxed{x=-2}$

$\boxed{P(-2,1)}$

Ck

$8(-2)-7(1) \stackrel{?}{=} -23$

$-16-7 \stackrel{?}{=} -23$

$-23=-23\checkmark$

Write and solve a system of equations for the following word problems:

9. At a recreation and sports facility, 3 members and 3 nonmembers pay a total of \$180 to take an aerobics class. A group of 5 members and 3 nonmembers pay \$210 to take the same class. How much does it cost members and nonmembers to take an aerobics class?

Let: m = member cost = \$15
n = non member cost = \$45

$$\begin{cases} 3m + 3n = 180 \\ 5m + 3n = 210 \end{cases} \rightarrow \begin{array}{r} 5m + 3n = 210 \\ -3m - 3n = -180 \\ \hline \frac{2m}{2} = \frac{30}{2} \end{array}$$

$m = 15$

$3(15) + 3n = 180$
$45 + 3n = 180$
$-45 \quad -45$
$\frac{3n}{3} = \frac{135}{3}$
$n = 45$

CK
$5(15) + 3(45) \stackrel{?}{=} 210$
$75 + 135 \stackrel{?}{=} 210$
$210 = 210$

10. One car model costs \$12,000 and costs an average of \$0.10 per mile to maintain. Another car model costs \$14,000 and costs an average of \$0.08 per mile to maintain. If one of each model is driven the same number of miles, after how many miles would the total cost be the same?

Let: x = number of miles = 100,000
y = cost of the vehicle = \$22,000

$$\begin{cases} y = .10x + 12{,}000 \\ y = .08x + 14{,}000 \end{cases}$$

$.10x + 12000 = .08x + 14000$
$-.08x - 12000 \quad -.08x - 12000$
$\frac{.02x}{.02} = \frac{2000}{.02}$
$x = 100{,}000$

11. The larger of two numbers is 5 more than twice the smaller. If the smaller is subtracted from the larger, the result is 12. Find the numbers.

Let: x = larger # = 19
y = smaller # = 7

$$\begin{cases} x = 2y + 5 \\ x - y = 12 \end{cases}$$

$2y + 5 - y = 12$
$y + 5 = 12$
$-5 \quad -5$
$y = 7$

$x - (7) = 12$
$+7 \quad +7$
$x = 19$

CK
$19 \stackrel{?}{=} 2(7) + 5$
$19 \stackrel{?}{=} 14 + 5$
$19 = 19$ ✓

12. On a canoe trip, Rita paddled upstream (against the current) at an average speed of 2 mi/h relative to the riverbank. On the return trip downstream (with the current), her average speed was 3 mi/h. Find Rita's paddling speed in still water and the speed of the river's current.

Let: p = paddling speed = 2.5 mph
c = rivers speed = .5 mph

$$\begin{cases} p - c = 2 \\ p + c = 3 \end{cases}$$

$$\frac{2p}{2} = \frac{5}{2}$$

$$p = 2.5$$

$$2.5 + c = 3$$
$$-2.5 \quad -2.5$$
$$c = .5$$

13. A light plane flew from its home base to an airport 255 miles away. With a head wind, the trip took 1.7 hours. The return trip with a tail wind took 1.5 hours. Find the average airspeed of the plane and the average windspeed.

Let: a = airplane's speed = 160 mph
w = wind speed = 10 mph

$$\begin{cases} a - w = \frac{255}{1.7} \\ a + w = \frac{255}{1.5} \end{cases} \rightarrow \begin{cases} a - w = 150 \\ a + w = 170 \end{cases}$$

$$\frac{2a}{2} = \frac{320}{2}$$

$$a = 160$$

$$(160) + w = 170$$
$$-160 \quad -160$$
$$w = 10$$

14. A bicycle store costs $2,400 per month to operate. The store pays an average of $60 per bike. The average selling price of each bicycle is $120. How many bicycles must the store sell each month to break even?

Let: b = # of bikes = 40
C = cost = $4800

$$\begin{cases} C = 2400 + 60b \\ C = 120b \end{cases}$$

$$120b = 60b + 2400$$
$$-60b \quad -60b$$
$$\frac{60b}{60} = \frac{2400}{60}$$
$$b = 40$$

$$C = 2400 + 60(40)$$
$$= 2400 + 2400$$
$$= \$4800$$

$$C = 120(40)$$
$$= \$4,800$$

15. The local zoo is filling two water tanks for the elephant exhibit. One water tank contains 50 gal of water and is filled at a constant rate of 10 gal/h. The second water tank contains 29 gal of water and is filled at a constant rate of 3 gal/h. When will the two tanks have the same amount of water? Explain.

let: h = # of hrs
t = gallons of water in the tank

$$\begin{cases} t = 10h + 50 \\ t = 3h + 29 \end{cases}$$

$3h + 29 = 10h + 50$
$-3h - 50 \quad -3h - 50$
$\frac{-21}{7} = \frac{7h}{7}$
$-3 = h$

$t = 10(-3) + 50$
$= -30 + 50$
$= 20$ gals

It's not possible the 1st tank has more water & is filling faster than the 2nd tank. They will never be the same.

16. At an ice cream parlor, ice cream cones cost \$1.10 and sundaes cost \$2.35. One day, the receipts for a total of one hundred seventy-two cones and sundaes were \$294.20. How many cones were sold?

let: i = # of ice cream cones = 88
s = # of sundaes = 84

$$\begin{cases} 1.10i + 2.35s = 294.20 \\ i + s = 172 \end{cases} \rightarrow$$

$1.10i + 2.35s = 294.20$
$-1.10i - 1.10s = -189.20$
$\frac{1.25s}{1.25} = \frac{105}{1.25}$
$s = 84$

$i + (84) = 172$
$-84 \quad -84$
$i = 88$

CK
$1.10(88) + 2.35(84) \stackrel{?}{=} 294.20$
$96.80 + 197.40 \stackrel{?}{=} 294.20$
$294.20 = 294.20$ ✓